Physics of switches

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Logic switches

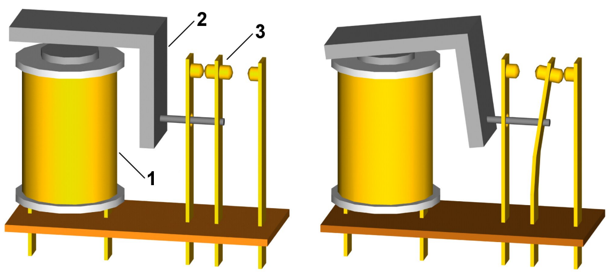
A *logic switch* is a device that can assume physically distinct states as a result of external inputs.

Usually the output of a physical system assume a continuous value (e.g. a voltage) and a threshold is used to partite the output space in two ore more states.

If the states are in the number of two we have binary logic switches: this is the case of transistors for modern microprocessors.

Computing with binary switches

Combinational switches can be easily employed do computation.
Sequential switches can be easily employed to store information.



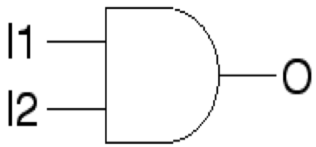
Conbinational:
in the absence of any external force, under equilibrium conditions, they are in the state S_0. When an external force F_{01} is applied, they switch to the state S_1 and remain in that state as long as the force is present. Once the force is removed they go back to the state S_0.

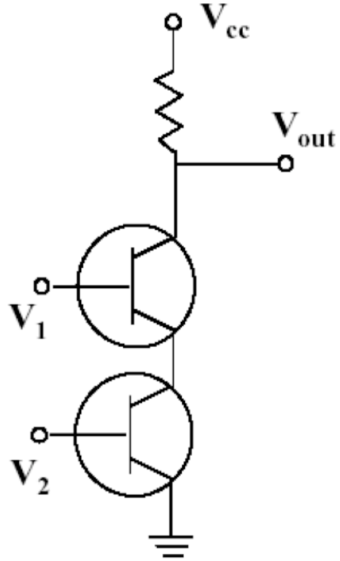
Sequential:
They can be changed from S_0 to S_1 by applying an external force F_{01}. Once they are in the state S_1 they remain in this state even when the force is removed. They go from S_1 to S_0 by applying a new force F_{10}. Once they are in S_0 they remain in this state even when the force is removed.

The switch

Logic gates are made by switches (presently transistors) and also memory cells can be represented in terms of switches.

I1	I2	O
0	0	0
0	1	0
1	0	0
1	1	1

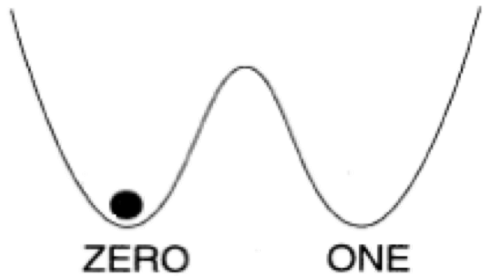




NAND gate

A physical model for the binary switch

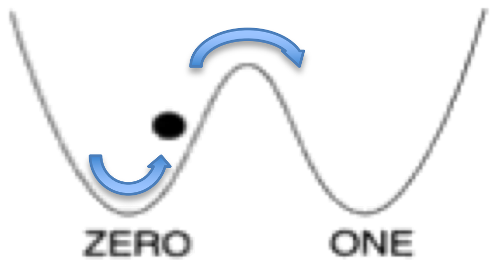
A simple switch can be represented by a physical dynamical model based on a bistable potential



We need a potential barrier in order to allow for physical distinguishability of the two states

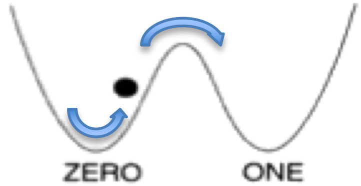
The switch

A simple switch can be represented by a physical dynamical model based on a bistable potential

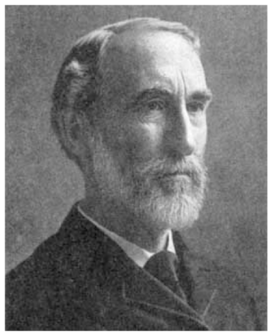


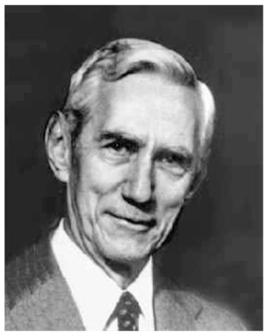
Switch event

Questions



- What is the minimum energy we have to spend if we want to produce a switch event ?
- Does this energy depends on the technology of the switch ?
- Does this energy depends on the instruction that we give to the switch ?
- ….

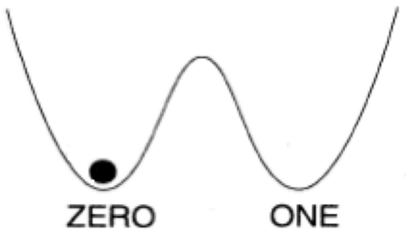


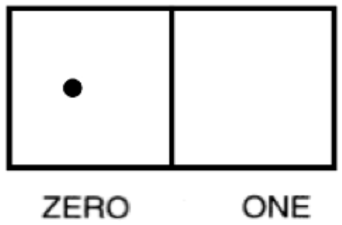


Some answers are still controversial…

The Physics of switches

In order to describe the physics of a switch we need to introduce a **dynamical model** capable of capturing the main features of a switch.





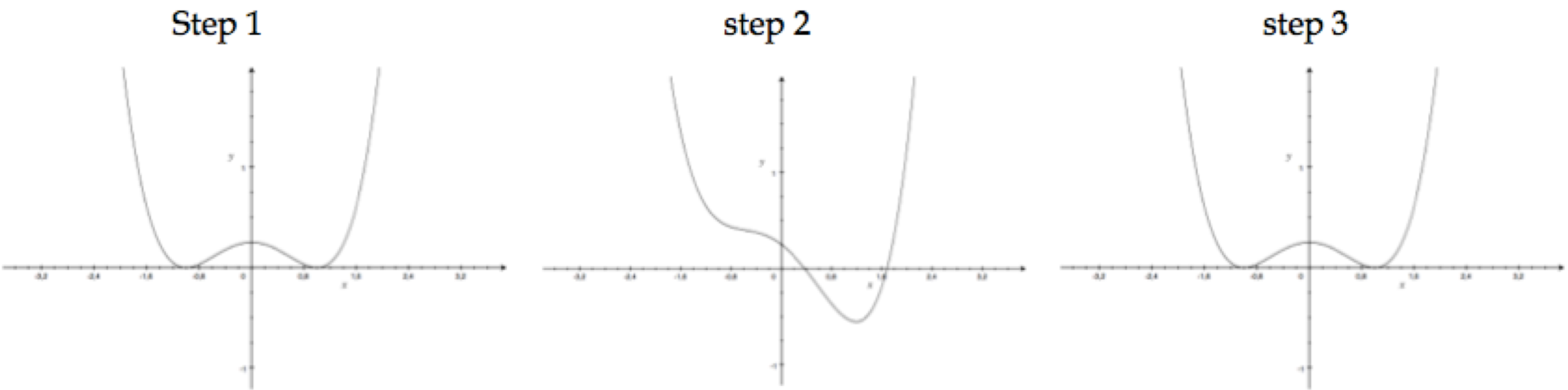
The two states, in order to be dynamically stable, are separated by some energy barrier that should be surpassed in order to perform the switch event.
This situation can be mathematically described by a second order differential equation like:

$$m\ddot{x} = -\frac{d}{dx}U(x) - m\gamma\dot{x} + F$$

The Physics of switches

According to this model if we want to produce a switch event we need to apply an external force *F* capable of bringing the particle from the left well (at rest at the bottom) into the right well (at rest at the bottom).
Clearly this can be done in more than one way.

As an example we start discussing what we call the **first procedure**: a three-step procedure based on the application of a large and constant force $F=-F_0$, with $F_0 > 0$



We can ask what is the minimum work that the force F has to perform in order to make the device switch from 0 to 1 (or equivalently from 1 to 0).
The work is computed as:

$$L = \int_{x_1}^{x_2} F(x)\,dx$$

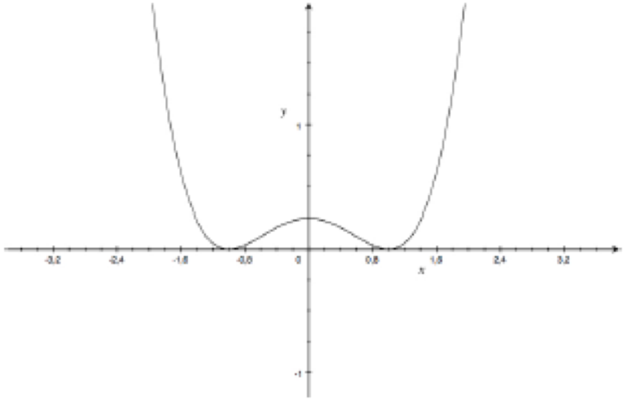
Thus $L = 2\ F_0$

The Physics of switches

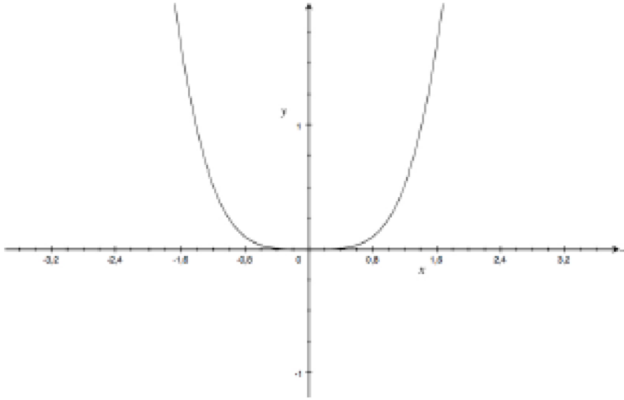
Is this the minimum work?

Let's look at this other procedure (**second procedure**):

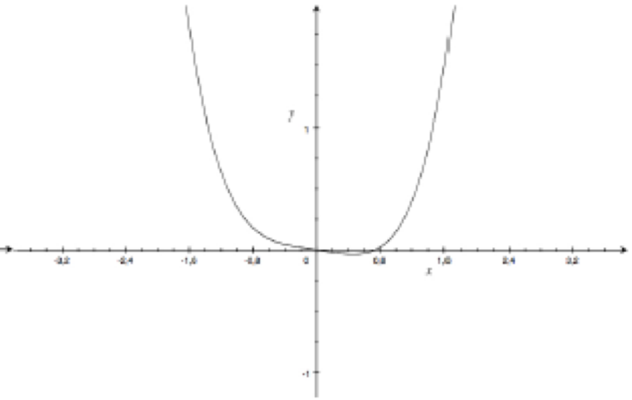
step 1



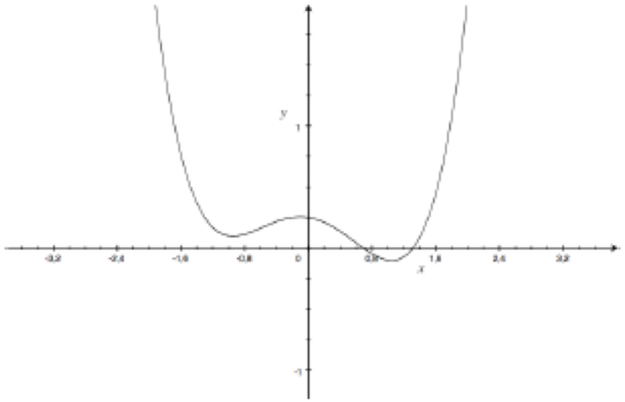
step 2



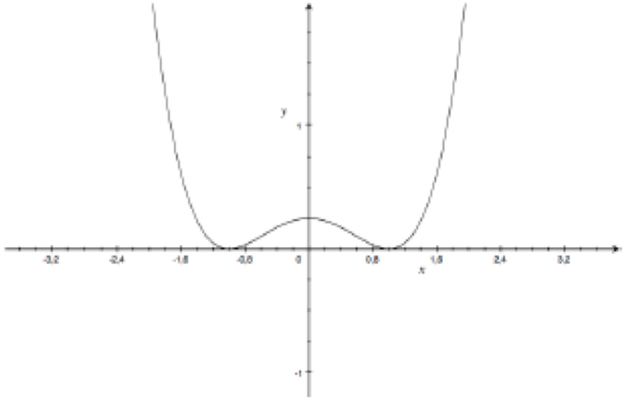
step 3



step 4



step 5



The only work performed happened to be during step 3 where it is readily computed as $L_1 = 2\ F_1$. Now, by the moment that $F_1 \ll F_0$ we have $L_1 \ll L_0$

The Physics of realistic switches

This analysis, although correct, is quite naïve, indeed. The reason is that we have assumed that the work performed can be made arbitrarily small.

IS THIS TRUE?

$\Delta U = L - Q$ Total energy variation during the switch

$\Delta U = 0 \longrightarrow L = Q$ and L=0

What about Q ?

The second principle of thermodynamics requires that: $Q \geq T\Delta S$

$$Q = T\,\Delta S + friction$$

We might be able to make friction = 0 but…what about entropy?

The Physics of realistic switches

Before looking for the system entropy, we need to check on the soundness of our model.

In order to be closer to a reasonable physical model, in fact, we need to introduce, together with a friction also a fluctuating force and thus a corresponding Langevin equation:

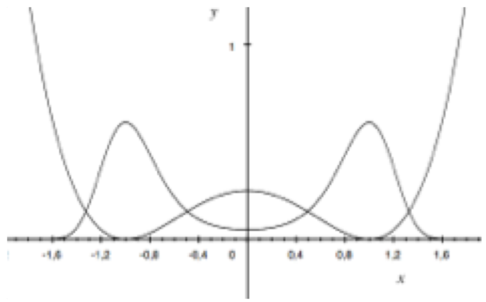
$$m\ddot{x} = -\frac{d}{dx}U(x) - m\gamma\dot{x} + \xi(t) + F$$

The relevant quantity becomes now the probability density *P(x,t)* and the probability

$$p_0(t) = \int_{-\infty}^{0} \boldsymbol{P(x,t)}dx \quad and \quad p_1(t) = \int_{0}^{+\infty} \boldsymbol{P(x,t)}dx$$

Represent the probability for our switch to assume “0” or “1” logic states

This calls for a reconsideration of the equilibrium condition that now is:



The Physics of realistic switches: the switch event

Based on these considerations we now define the switch event as the transition from an initial condition toward a final condition, where the initial condition is defined as $<x> < 0$ and the final condition is defined as $<x> > 0$. With the initial condition characterized by:

$$p_0(t) = \int_{-\infty}^{0} \boldsymbol{P}(\boldsymbol{x}, \boldsymbol{t})dx \cong 1 \quad and \quad p_1(t) = \int_{0}^{+\infty} \boldsymbol{P}(\boldsymbol{x}, \boldsymbol{t})dx \cong 0$$

and the final condition by:

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In order to produce the switch event we proceed as follows: we set our initial position at any value $x < 0$ and wait a time t_a, with $\tau_1 << t_a << \tau_2$, then we apply an external force *F* for a time t_b in order to produce a change in the $<x>$ value from $<x> < 0$ to $<x> > 0$. Then we remove the force. In practice we need to wait a time t_a after the force removal in order to verify that the switch event has occurred, i.e. that $<x> > 0$. The total time spent has to satisfy the condition $2\,t_a + t_b << \tau_2$.

Now that we have defined the switch event in this new framework, we can go back to our question: what is the minimum energy required to produce a switch event?

The Physics of realistic switches: the role of entropy

In this new physical framework we have to do with exchanges of both work and heat (constant temperature transformation approximation).
Thus we have to take into account both the exchanges associate with work and the changes associated with entropy variation.

Entropy here is defined according to Gibbs: $S = -K_B \int_{-\infty}^{+\infty} p(x) \ln(p(x))\, dx$

Now the question is: what happens to the entropy during the switch?

The Physics of realistic switches

The previous analysis, although correct, is quite naïve, indeed.
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IS THIS TRUE?
In a macroscopic physical system, we have:

$\Delta U = L - Q$ Total energy variation during the switch

$\Delta U = 0$ → $L = Q$ and L=0

What about Q ?

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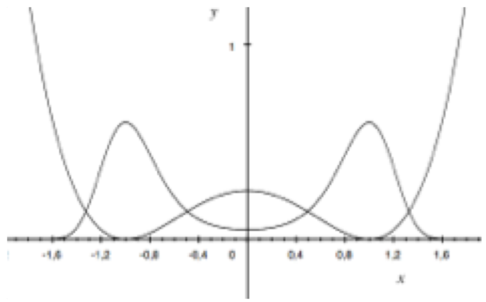

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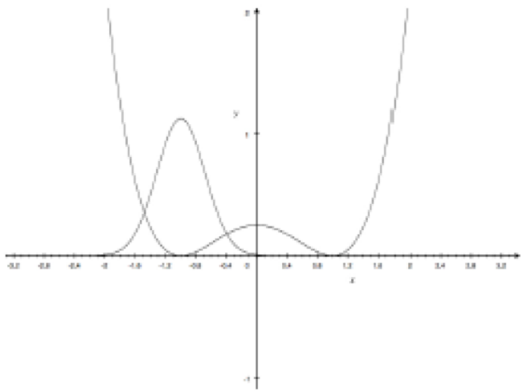
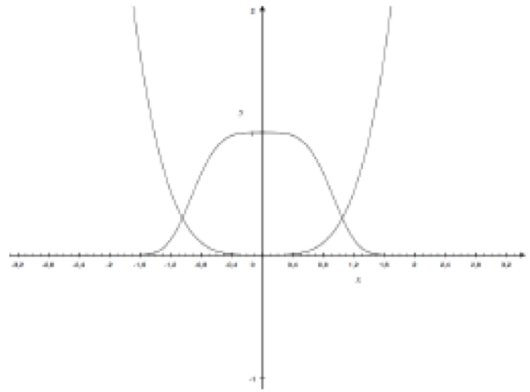
The Physics of realistic switches

Based on this new approach let's review the previous procedure:

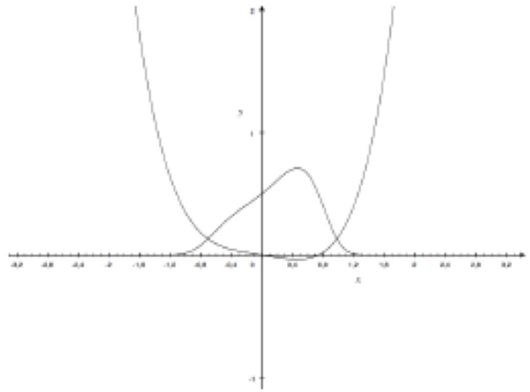
step 1



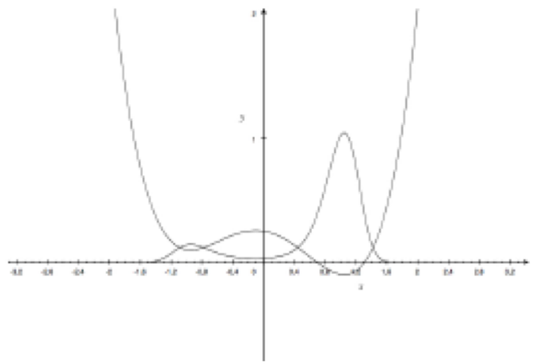
step 2



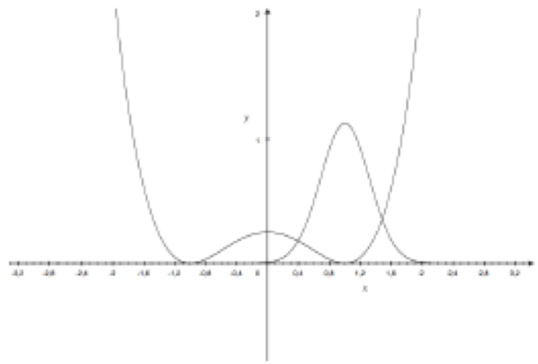
step 3



step 4



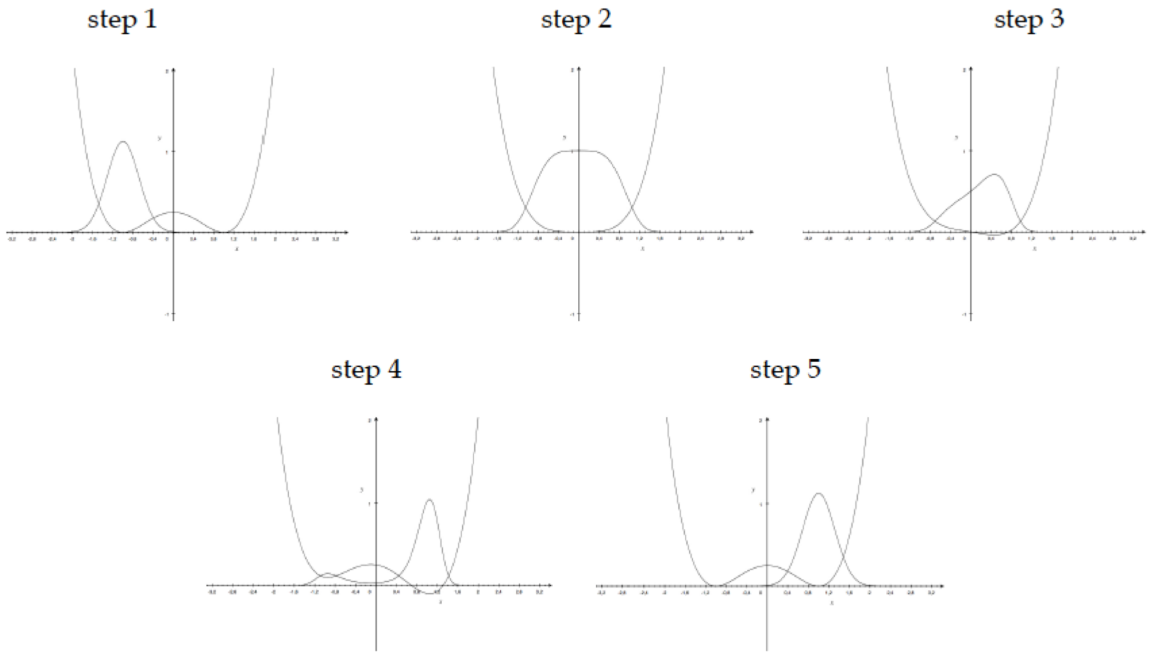
step 5



The total change of entropy is represented by $S_5 - S_1 = 0$ by the moment that S_1=S_5.

However it is important to realize that the change step 1→step 2 is most probably realized as a free expansion, and thus it results in a non equilibrium entropy increase (non reversible).

The Physics of realistic switches



Thus if we allow for an **out-of equilibrium** entropy increase we will have a problem when the entropy should decrease back in steps 3→ 4 → 5.

In practise, the change step 1→ 2 with a free expansion is free (entropy increses, see the Joule experiment on free expansion) but the entropy decrease to come to the final state will require an entropy drecrease and this will require an energy expenditure.
Thus $\Delta S < 0$ will require a $Q > 0$ and thus a non-null energy.

The Physics of realistic switches

Based on these considerations we can now reformulate conditions required in order to perform the switch by spending zero energy:

1) The total work performed on the system by the external force has to be zero.
2) The switch event has to proceed with a speed arbitrarily small in order to have arbitrarily small losses due to friction.
3) The system entropy **never decreases** during the switch event **or** the increase-decrease is performed at equilibrium (no free expansion).

Is it possible?